

Haldia Institute of Technology
Department of Applied Science

Assignment - I

Course: PH 301/PH 401

Module 1: Vector Calculus

1. Define vector triple product and scalar triple product?
2. State and explain the Gauss divergence and Stokes theorem.
3. Explain the physical significance of divergence and curl of a vector.
4. Define curl of a vector and give its physical significance.
5. Define Scalar field and vector field function with examples?
6. Define solenoidal and irrotational vector.
7. Show that the divergence of a curl of a vector is zero.
8. Show that curl of a gradient of a scalar is zero.
9. If a vector **A** and **B** are irrotational then show that the vector **A**×**B** is solenoidal.
10. Write the expression for curl, divergence and gradient in spherical, cylindrical and Cartesian co ordinate system.
11. The two vectors **A**=3x**j**+4y**j**+5z**k** and **B**=x²**j**+4y**j**+z³**k**
Find the angle between them
Find their cross product **A**×**B** and **A**·**B**
Find curl and divergence of **A** and **B** vector.
12. Show that vector field **A**=(2x-yz)**i**+(2y-zx)**j**+(2z-xy)**k** is irrotational and conservative.
13. Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at (1,-2,-1) along the direction $2\hat{i} - \hat{j} - 2\hat{k}$
14. If the potential of a field is given by $V(x,y,z)=(4x^2+2y^2+z^2)^{1/2}$ find the field intensity at the point (1,1,1)

Additional assignment:

EXERCISES

- 1 Prove that the geometric interpretation of a cross product is as an area.
- 2 Start from a picture of $\vec{C} = \vec{A} - \vec{B}$ and use the definition and properties of the dot product to derive the law of cosines. (If this takes you more than about three lines, start over, and no components, just vectors.)
- 3 Start from a picture of $\vec{C} = \vec{A} - \vec{B}$ and use the definition and properties of the cross product to derive the law of sines. (If this takes you more than a few lines, start over.)
- 4 Show that $\vec{A} \cdot \vec{B} \times \vec{C} = \vec{A} \times \vec{B} \cdot \vec{C}$. Do this by drawing pictures of the three vectors and find the geometric meaning of each side of the equation, showing that they are the same (including sign).
- 5 (a) If the dot product of a given vector \vec{F} with every vector results in zero, show that $\vec{F} = 0$.
(b) Same for the cross product.
- 6 From the definition of the dot product, and in two dimensions, draw pictures to interpret $\vec{A} \cdot (\vec{B} + \vec{C})$ and from there prove the distributive law: $\vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$. (Draw $\vec{B} + \vec{C}$ tip-to-tail.)
- 7 For a sphere, from the definition of the integral, what is $\oint d\vec{A}$? What is $\oint dA$?
- 8 What is the divergence of $\hat{x}xy + \hat{y}yz + \hat{z}zx$?
- 9 What is the divergence of $\hat{r}r \sin \theta + \hat{\theta}r_0 \sin \theta \cos \phi + \hat{\phi}r \cos \phi$? (spherical)
- 10 What is the divergence of $\hat{r}r \sin \phi + \hat{\phi}z \sin \phi + \hat{z}zr$? (cylindrical)
- 11 In cylindrical coordinates draw a picture of the vector field $\vec{v} = \hat{\phi}r^2(2 + \cos \phi)$ (for $z = 0$). Compute the divergence of \vec{v} and indicate in a second sketch what it looks like. Draw field lines for \vec{v} .
- 12 What is the curl of the vector field in the preceding exercise (and indicate in a sketch what it's like).
- 13 Calculate $\nabla \cdot \hat{r}$. (a) in cylindrical and (b) in spherical coordinates.
- 14 Compute $\partial_i x_j$.
- 15 Compute $\text{div curl } \vec{v}$ using index notation.
- 16 Show that $\epsilon_{ijk} = \frac{1}{2}(i - j)(j - k)(k - i)$.
- 17 Use index notation to derive $\nabla \cdot (f\vec{v}) = (\nabla f) \cdot \vec{v} + f\nabla \cdot \vec{v}$.